Do we know what a black hole is? A conceptual refinement

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A modern observatory in the land of ancient observatories

Four Pillars of Relativistic Astrophysics

- Supernovae;
- Neutron Stars;
- Black Holes;
- GRBs.

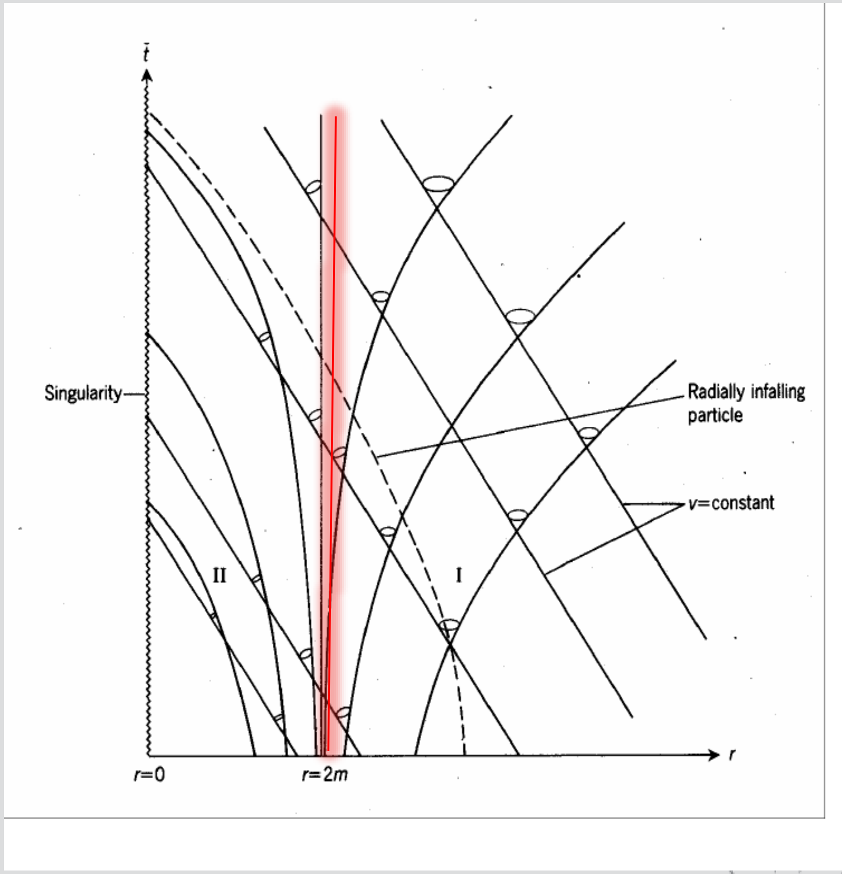
Escape velocity and Horizon

- John Michel: 1783
- Laplace: 1796

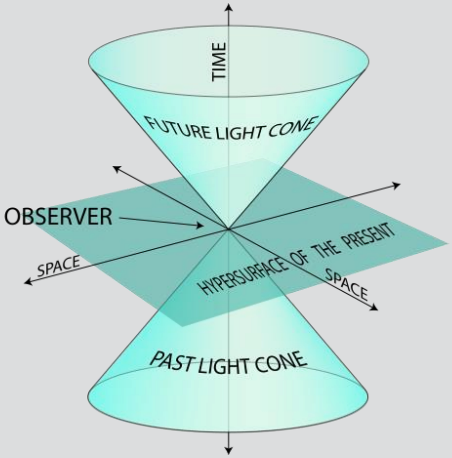
Escape velocity may exceed light velocity

Karl Schwarzschild

- Karl Schwarzschild 1916
- David Finkelstein 1958:



Event Horizon as a one sided membrane



Typology of BHs

- BH in Field Theory
- PBH
  - Observational Constraint
- CBH
  - Model Building
  - Different to Stationary ABHs
  - Observational Consequences
- ABH
  - Astrophysical Implications
  - Observational Evidence

ABHs versus CBHs

ABH:

- Vacuum Einstein Equations;
- Asymptotically Minkowskian;
- Stationary/Static;
- Spherically Symmetric.

Cosmological Black Holes:

- Non-vacuum Cosmological Solutions;
- Inhomogeneous asymtotically FRW ;
- Non-stationary;
- Spherically Symmetric.

- Mc Vittie metric (1933)

$$ds^2 = -\left(\frac{1-\frac{M}{2N}}{1+\frac{M}{2N}}\right)^2 dt^2 + e^{\beta(t)}\left(1+\frac{M}{2N}\right)^4 (dr^2 + h^2 d\Omega^2)$$

$$h(r) = \begin{cases} \sinh(r) & k=1 \\ r & k=0 \\ \sin(r) & k=-1 \end{cases}$$

$$N(r) = \begin{cases} 2\sinh(r/2) & k=1 \\ r & k=0 \\ 2\sin(r/2) & k=-1 \end{cases}$$

$$M = me^{\beta(t)/2}$$

Representing a point mass with a weak singularity at

$$e^{\beta/2}\left(1+\frac{M}{2N}\right)^2 h = 2M$$

Not suitable as a model for a cosmological black hole!

- Sultana-Dyer metric

$$ds^2 = t^4[(1-\frac{2m}{r})dt^2 - \frac{4m}{r}dtdr - (1-\frac{2m}{r})dr^2 - r^2 d\Omega^2]$$

Some pathological Behaviors:

- It has a *conformal Killing horizon;*
- Petrov type D;
- Containin a non-comoving two-fluid source: one is dust and the other is a null fluid;
- The cosmological fluid becomes tachyonic at late times near the horizon
- . Not suitable as a model for a cosmological black hole!

LTB metric

- Inhomogeneous cosmological solution;
- Spherically symmetric;
- Pressure-less ideal fluid.

$$ds^2 = -dt^2 + \frac{R'^2}{1+f} dr^2 + R^2 d\Omega^2,$$

- *Metric functions f(r) and R(r, t);*
- *RW-like coordinates (FRW: f=0, R =r.a(t))*

LTB Solution of Einstein Equations

$$R = -\frac{M}{f}(1-\cos(\eta(r,t))),$$

$$\eta - \sin(\eta) = \frac{(-f)^{3/2}}{M}(t - t_n(r)),$$

for $f < 0$, and

$$R = (\frac{9}{2}M)^{\frac{1}{3}}(t - t_n)^{\frac{2}{3}},$$

for $f = 0$, and

$$R = \frac{M}{f}(\cosh(\eta(r,t)) - 1),$$

$$\sinh(\eta) - \eta = \frac{f^{3/2}}{M}(t - t_n(r)),$$

$$M = \frac{1}{2}\int_0^R \rho R^2 dR$$

$$\rho = \frac{2M'}{R^2 R'},$$

Integration constants: $M(r), t_b(r)$

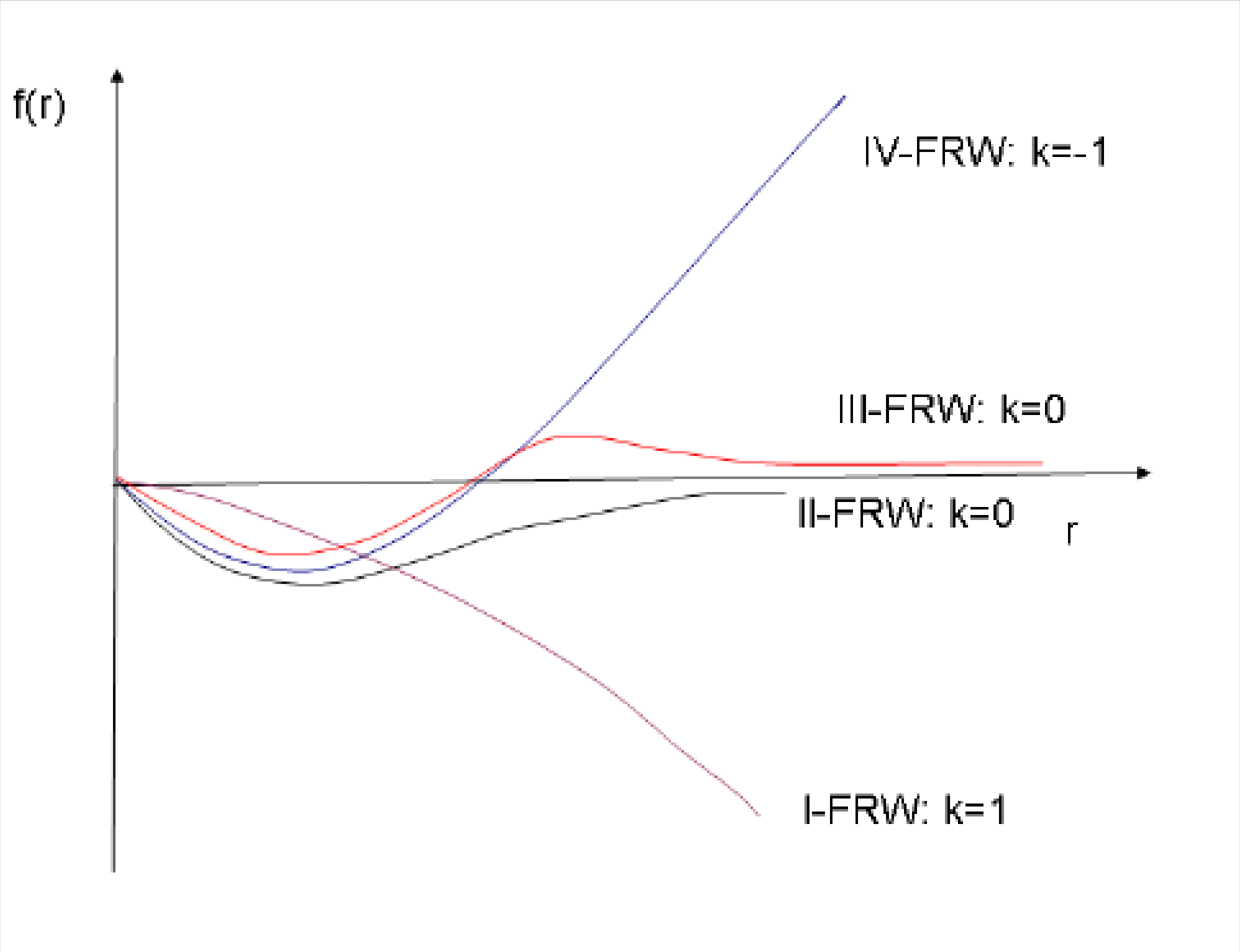
How to use LTB metric?

How to construct a cosmological structure?

Spherically symmetric cosmological structure

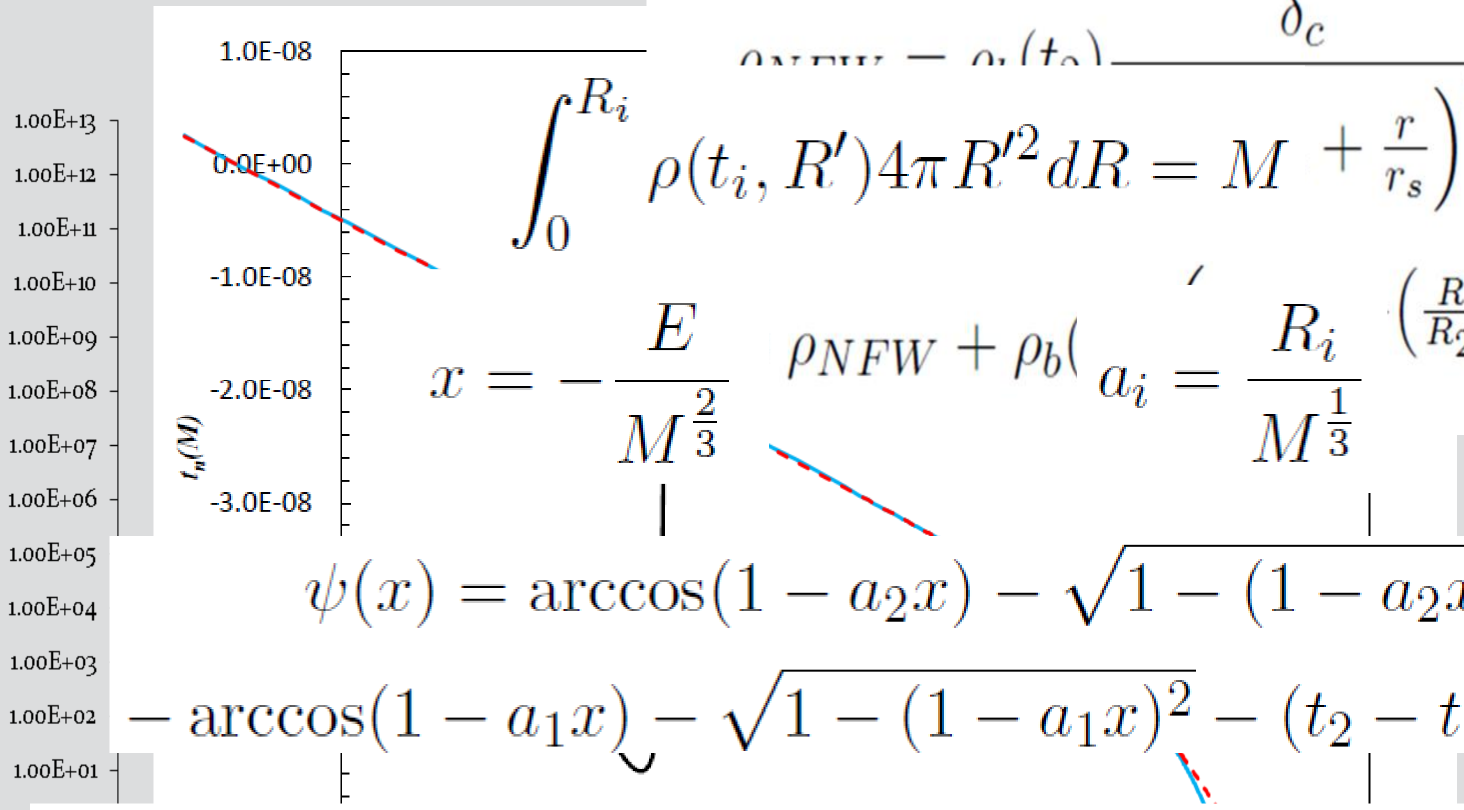
- The simplest case of a structure embedded in an expanding cosmological environment!
- More realistic than an internal Schwarzschild solution;
- Although the density outside the structure is negligible (cosmic), we may encounter counter-intuitive effects;
- Excellent arena to test effects due to very weak gravity yet general relativistic at large;
- Test of quasi-local effects (far from the range of validity of EP).

Behavior of Curvature Function f



J. T. Firouzjaee, Reza Mansouri, Gen. Relativity Gravitation. 42, 2431 (2010)

Making a Structure in LTB model



$$\rho_{NFW} = \rho_b(t_0)\frac{\delta_c}{\ldots \left(\ldots + \frac{r}{r_s}\right)^2}$$

$$\int_0^{R_i} \rho(t_i, R')4\pi R'^2 dR = M$$

$$x = -\frac{E}{M^{\frac{2}{3}}} \qquad \rho_{NFW} + \rho_b(\qquad a_i = \frac{R_i}{M^{\frac{1}{3}}} \qquad \left(\frac{R}{R_2}\right)^2$$

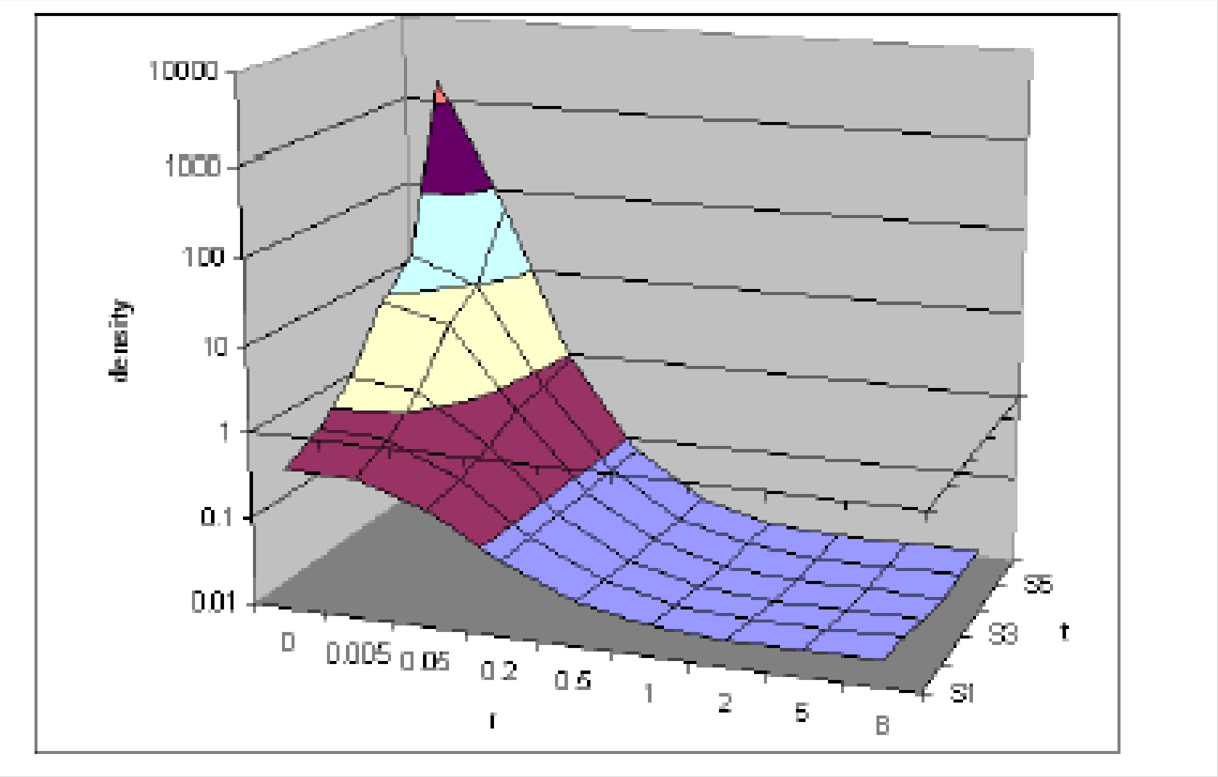
$$\psi(x) = \arccos(1 - a_2 x) - \sqrt{1 - (1 - a_2 x)^2}$$

$$- \arccos(1 - a_1 x) - \sqrt{1 - (1 - a_1 x)^2} - (t_2 - t_1)x^{\frac{3}{2}} = 0$$

$$t_b = t_i - \frac{M}{E^{\frac{3}{2}}}\left[\arccos\left(1 + \frac{ER_i}{M}\right) - \sqrt{1 - \left(1 + \frac{ER_i}{M}\right)^2}\right]$$

Krasinski-Hellaby PRD (65) 023501 (2001)

Formation of CBH in a Flat FRW



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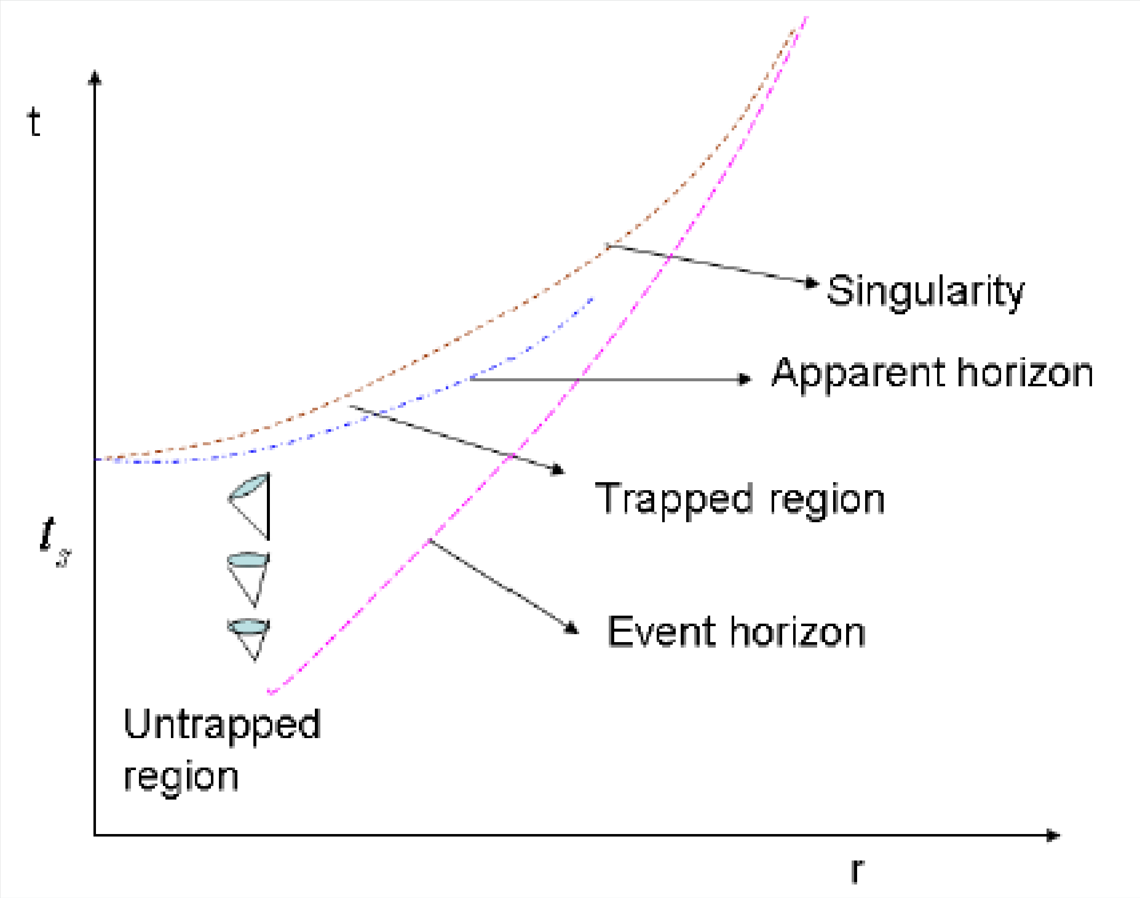
BH in Expanding Universe



Dynamical Horizon: Trapped Region

Definition 1 [8]. A *trapping horizon* H is a hypersurface in a 4-dimensional spacetime that is foliated by 2-surfaces such that $\theta_{(\ell)} \; |_H = 0$, $\theta_{(n)} \; |_H \neq 0$, and $\pounds_n \theta_{(\ell)} \; |_H \neq 0$. A trapping horizon is called *outer* if $\pounds_n \theta_{(\ell)} \; |_H < 0$, *inner* if $\pounds_n \theta_{(\ell)} \; |_H > 0$, *future* if $\theta_{(n)} \; |_H < 0$, and *past* if $\theta_{(n)} \; |_H > 0$. The most relevant case in the context of black holes is the *future outer trapping horizon*(FOTH).

Weakly Isolated Horizon

Definition 2 [9]. A *weakly isolated horizon* is a three-surface H such that :

1. H is null;
2. The expansion $\theta_{(\ell)}\,|_H = 0$ where ℓ^a, being null and normal to the foliations S of H;
3. $-T_a^b \ell^a$ is future directed and causal;
4. $\pounds_\ell \omega_a = 0$, where $\omega_a = -n_b \nabla_{\underleftarrow{a}} \ell^b$, and the arrow indicates a pull-back to H.

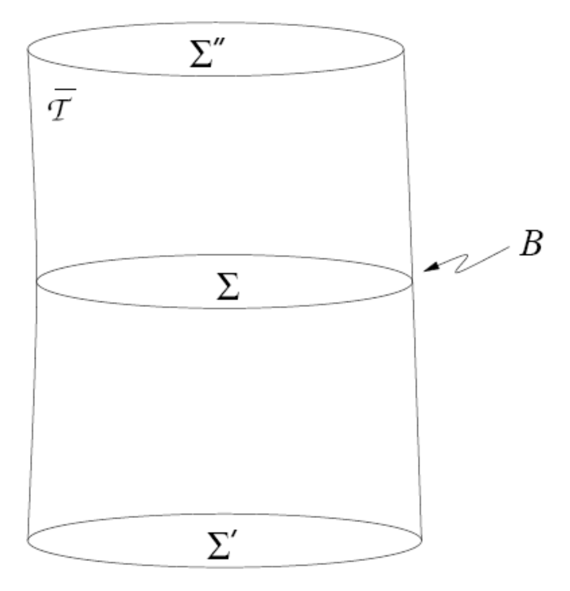
What is the mass of the structure?

Quasi-local Mass in GR

- No local definition of mass;
- No unique definition of mass;
- Misner-Sharp is the most simple one in the case of spherically symmetric overdensities!

2+1 dimensional hypersurface encompassing the structure

Note the freedom to add terms depending on the fixed boundary data (reference term = zero point energy)



$$S[g] = \frac{1}{2\kappa}\int_M d^4x\sqrt{-g}(\Re - 2\Lambda) + \frac{1}{\kappa}\int_{t'}^{t''} d^3x\sqrt{h}K - \frac{1}{\kappa}\int_{^3B} d^3x\sqrt{-\gamma}\Theta - S^0$$

Brown-York & Liu-Yau Mass

$$E = \frac{1}{8\pi} \oint_B d^2x \sqrt{\sigma} (k - k_0)$$

- Brown-York mass

$$E = -\frac{1}{8\pi} \oint_B d^2x \sqrt{\sigma} \left(\sqrt{k^2 - \ell^2} - k_0 \right)$$

- Liu-Yau mass

Hawking-Hayward Mass

$$E_H(S) = \sqrt{\frac{Area(S)}{16\pi G^2}}\left(1 + \frac{1}{2\pi}\oint_S \rho\rho' dS\right)$$
$$= \sqrt{\frac{Area(S)}{16\pi G^2}}\left(\oint (-\Psi_2 - \sigma\lambda + \Phi_{11} + \Lambda) dS\right)$$

$$E_H(S) = \sqrt{\frac{Area(S)}{16\pi G^2}}\left[1 + \frac{1}{2\pi}\oint_S \left(\rho\rho' - \frac{1}{8}\sigma_{ab}\overline{\sigma}^{ab} - \frac{1}{2}\omega_a\omega^a\right) dS\right]$$

- **Misner-Sharp mass**

In general spherical symmetric case

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R(t,r)^2 d\Omega^2.$$

$$\rho = \frac{2M'}{R^2R'}\ , \quad p_r = -\frac{2\dot{M}}{R^2\dot{R}},$$

$$M = \frac{1}{2}\int_0^R \rho R^2 dR.$$

$$M = \frac{1}{8\pi}\int_0^r \rho\sqrt{(1+(\frac{dR}{d\tau})^2 - \frac{2M}{R})}d^3V.$$

It becomes **ADM** and **BS** in asymptotically flat space time.

LTB metric

$$M_{Hawking-Hayward} = M_{Misner-Sharp} = M(r)$$

- r=Const.

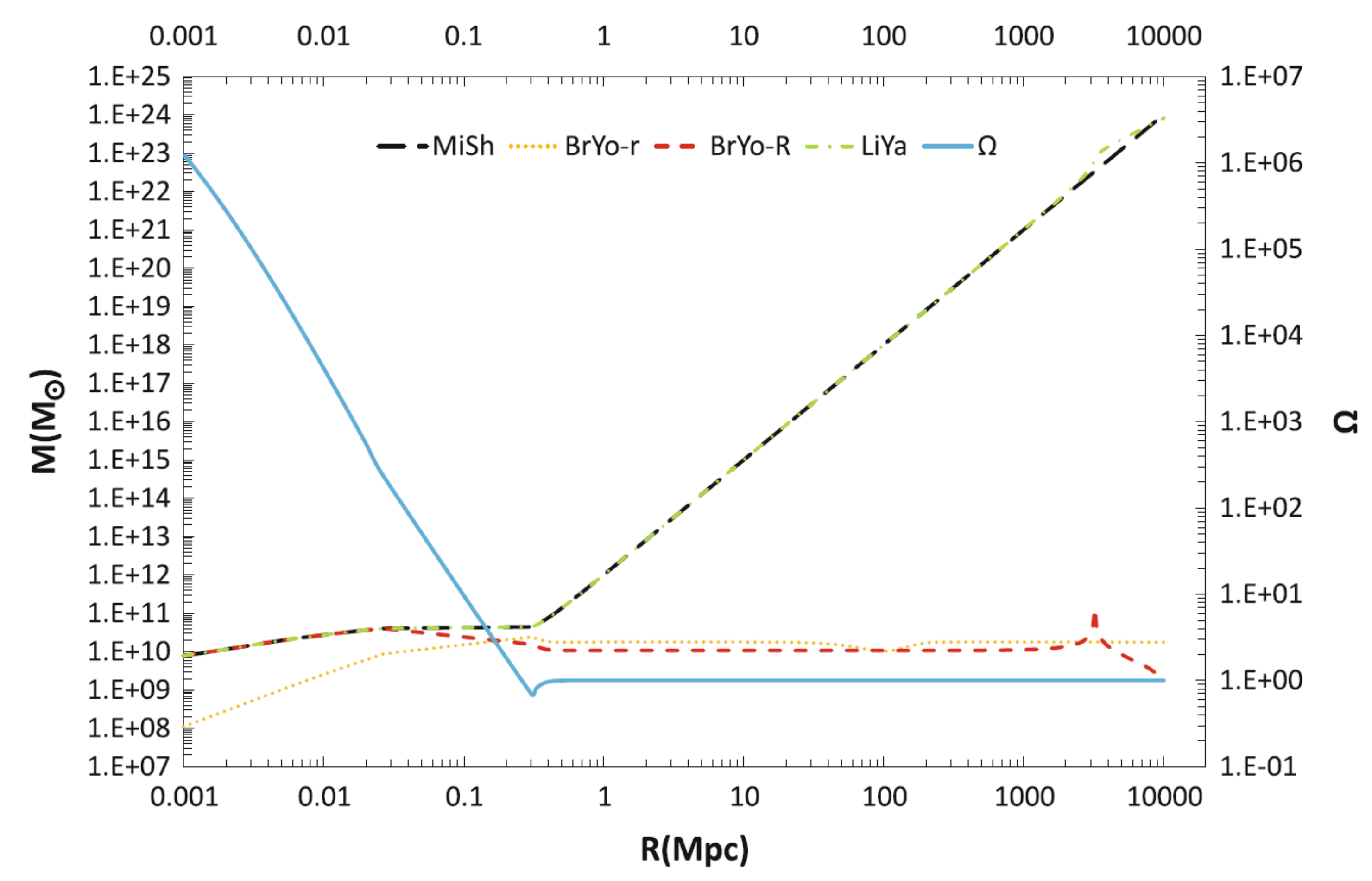
$$M_{\mathrm{BY}} = -R\sqrt{1+f} - \text{Subtraction term.}$$

- R=Const.

$$M_{\mathrm{BY}}|_{R=const} = -R\sqrt{1-\frac{2M}{R}} - \text{Subtraction term.}$$

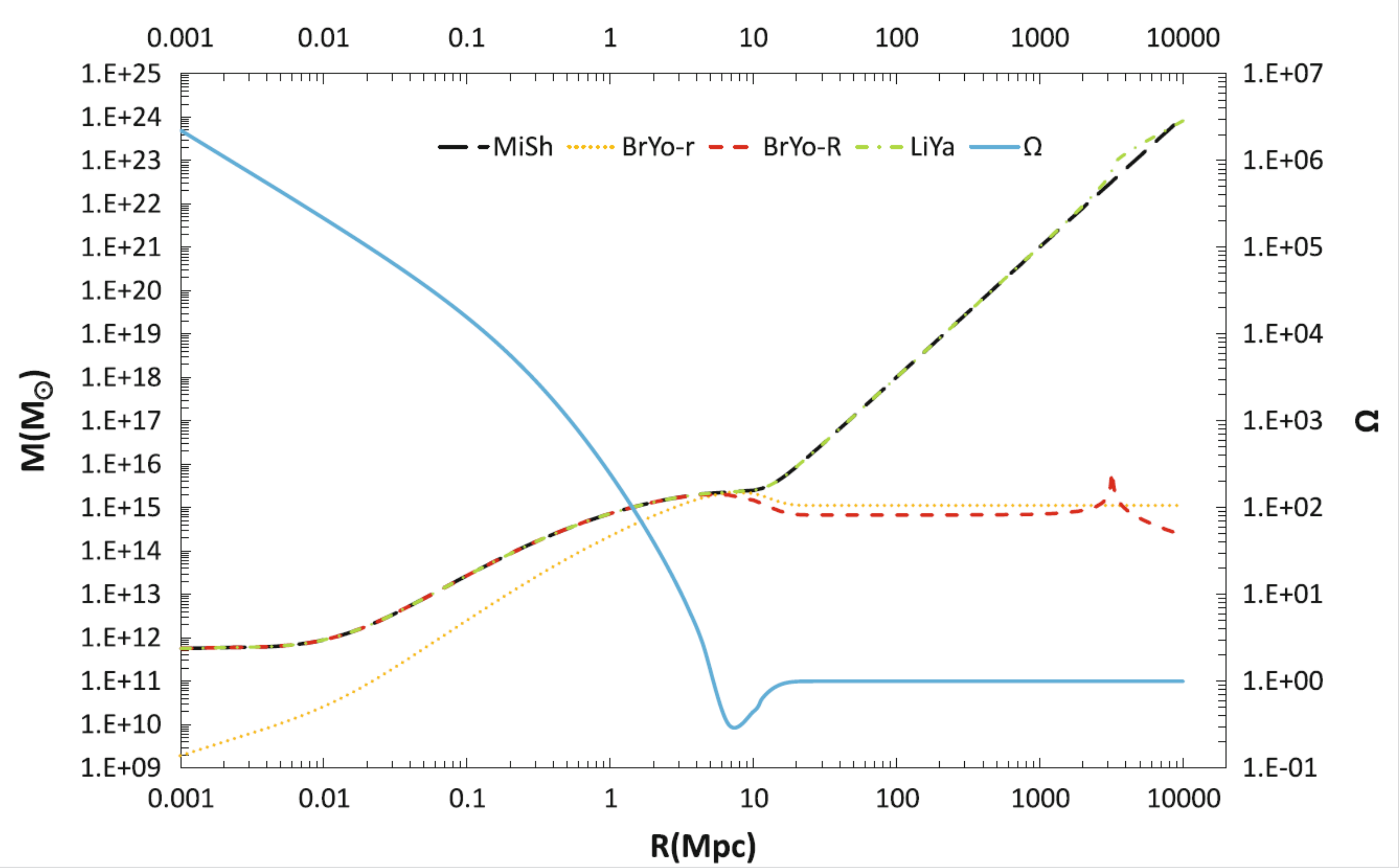
$$M_{\mathrm{LY}} = -R\sqrt{1-\frac{2M}{R}} - \text{Subtraction term,}$$

Different Masses for a Galaxy with NFW Density Profile



J. T. Firouzjaee, M. Parsi Mood and Reza Mansouri, Gen. Relativ. Gravit. 44, 639 (2012)

Different Masses for a Cluster of Galaxies with NFW Density Profile



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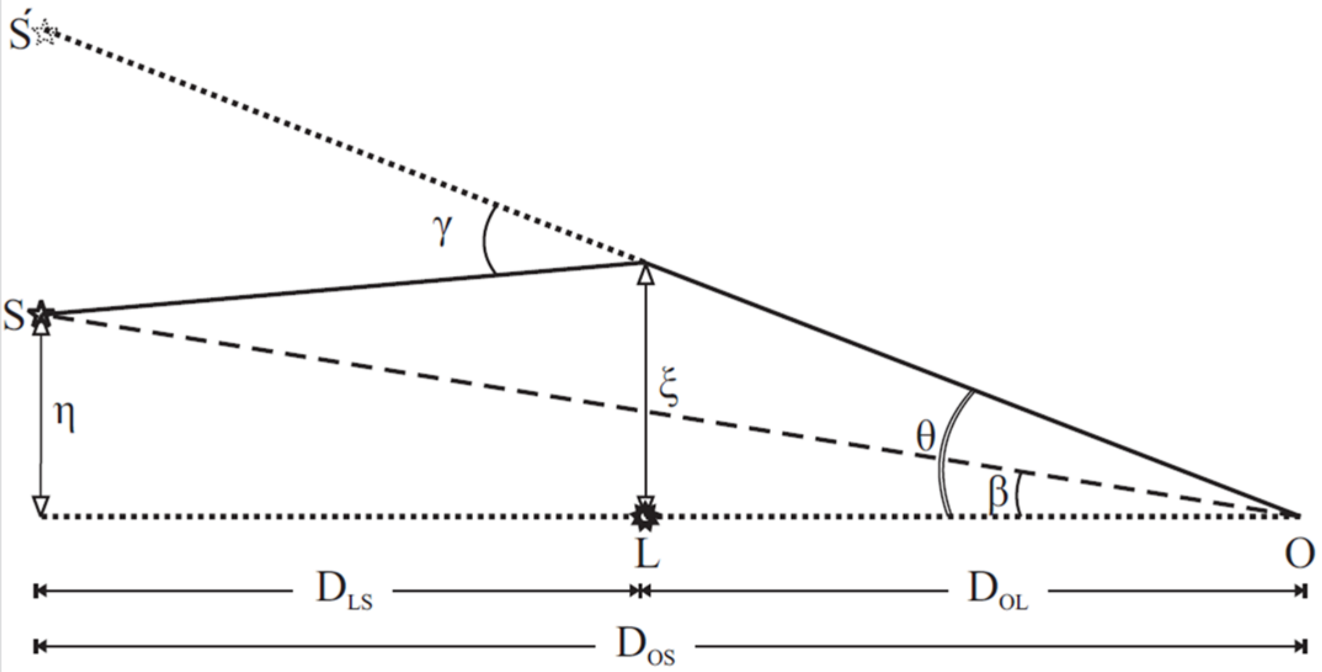
How can the simple problem of light deflection be solved?

Light deflection across a dynamical cosmic structure

Simple Geometry of Light Deflection by a Dynamical Structure



$$\gamma = (\theta - \beta)\frac{D_{OS}}{D_{LS}}$$

M. Parsi Mood, J. T. Firouzjaee, Reza Mansouri

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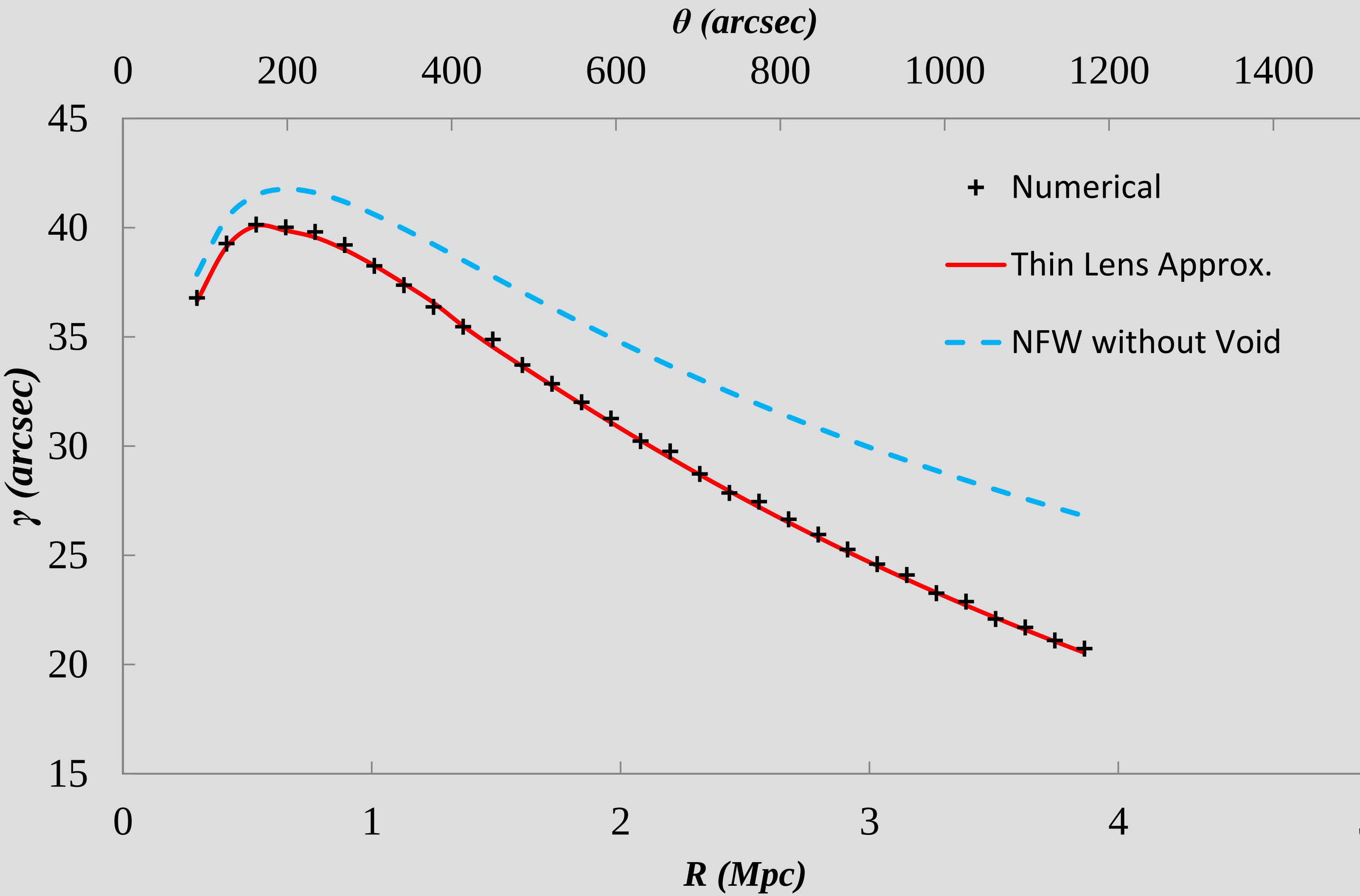
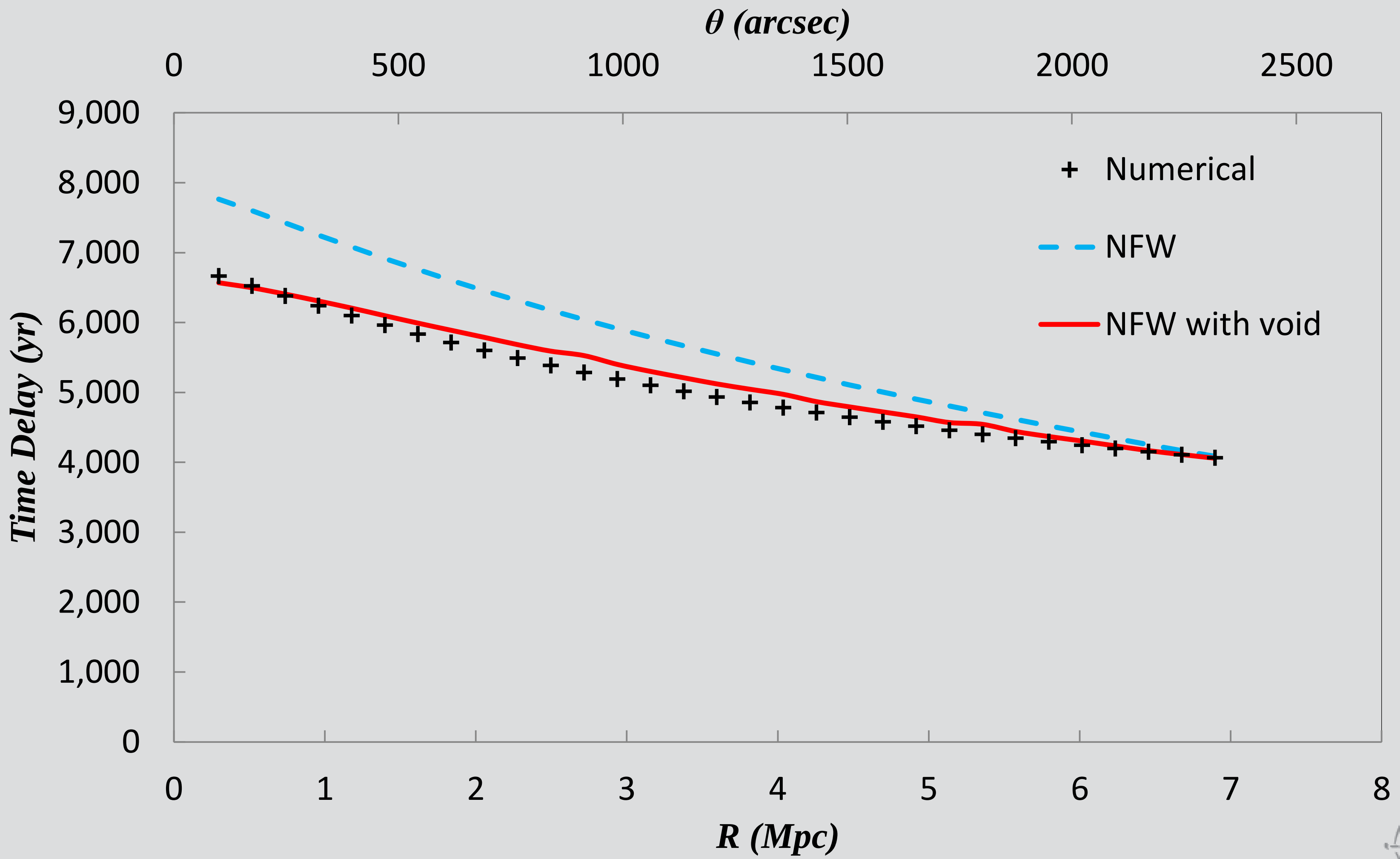
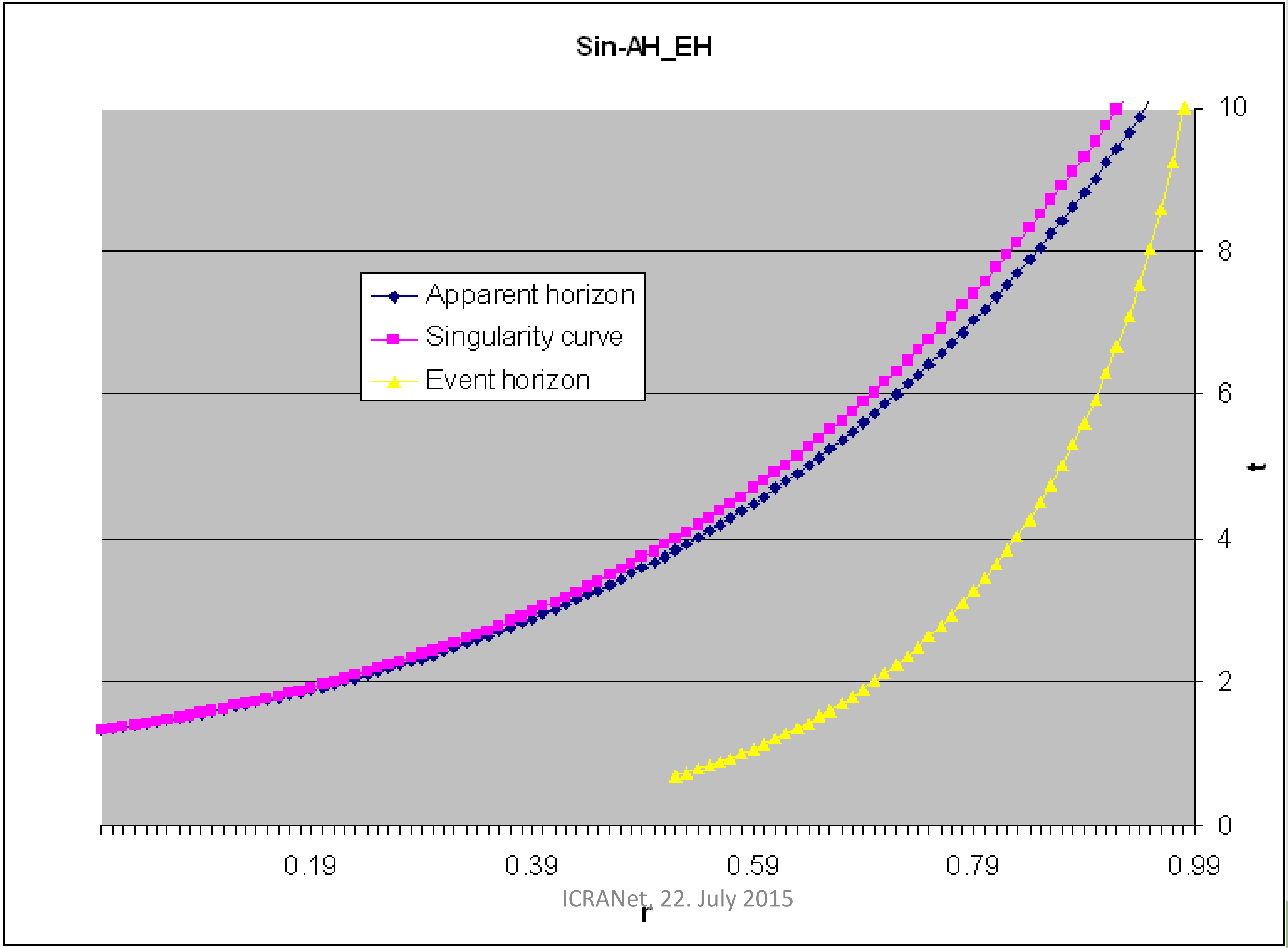
Deflection Angle



Delay time (Between lensed and unlensed)




Sin-AH_EH
Apparent horizon
Singularity curve
Event horizon
10
8
6
4
2
0
t
0.19
0.39
0.59
0.79
0.99
r

equations is a delicate issue. We first started with the familiar Runge-Kutta adaptive step size algorithm with proportional and integral feedback (PI control) [19] in which the step size is adjusted to keep local error under a suitable threshold. We started with the so-called embedded Runge-Kutta of the rank 5(4). It turned out, however, that its accuracy is too low. Therefore, we tried the rank 8(7) and then the rank 11(10) algorithm. The difference between these two last ranks, however, turned out to be marginal and below one percent. Given the time-consuming rank 11(10) algorithm, we preferred to use the rank 8(7) one. Now, as a fist test for the

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expecting a null result. The result was a non-negligible deflection angle of the order of few milliarcseconds. Suspecting to face a numerical effect, and trying to understand the numerical algorithm and the source of this numerical effect, we continued to calculate a more concrete and non-trivial LTB case. The result for the rank 8(7) Runge-Kutta numerical method applied to a structure with a compact density profile did agree with the thin lens approximation. However, in the case of a more diffuse density profile the result showed a deflection angle up to an order of magnitude higher than the thin lens approximation. We did interpret this result as a sign not to trust the Runge-Kutta method and turned to an alternative numerical method!

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gave an acceptable null result. We, therefore, decided to integrate our geodesic equations using the semi-implicit Rosenbrock method instead of the Runge-Kutta one.

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Collapse of a structure with pressure

Spherically symmetric structure in a cosmological background filled with perfect fluid with non-vanishing pressure as an exact solution of Einstein equations using the Lemaitre solution:

Rahim Moradi, Javad T. Firouzjaee, Reza Mansouri: arXiv:1504.04746

- http://arxiv.org/abs/1503.05020
- http://arxiv.org/abs/1408.0778

Two papers by Javad T. Firouzjaee and George F. Ellis

http://arxiv.org/abs/1408.0778

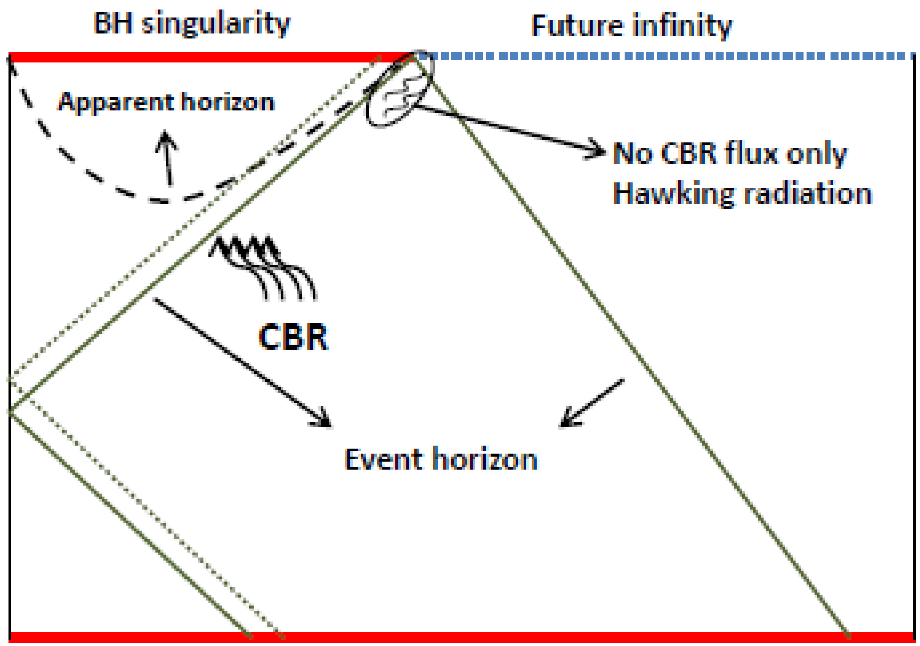

FIG. 5: *Penrose diagram for a black hole in a ΛCDM expanding universe, where there is a matter and radiatio influx, but it dies away to zero in the far future. Note that we do not represent back reaction effects in this picture*